

Effect of spatial bias on the nonequilibrium phase transition in a system of coagulating and fragmenting particles

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We examine the effect of spatial bias on a nonequilibrium system in which masses on a lattice evolve through the elementary moves of diffusion, coagulation, and fragmentation. When there is no preferred directionality in the motion of the masses, the model is known to exhibit a nonequilibrium phase transition between two different types of steady state, in all dimensions. We show analytically that introducing a preferred direction in the motion of the masses inhibits the occurrence of the phase transition in one dimension, in the thermodynamic limit. A finite-size system, however, continues to show a signature of the original transition, and we characterize the finite-size scaling implications of this. Our analysis is supported by numerical simulations. In two dimensions, bias is shown to be irrelevant.

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I. INTRODUCTION

Systems far from equilibrium can undergo nonequilibrium phase transitions between two different types of steady state when the parameters of the system are varied. It is important to know how robust such transitions are with respect to changes in the governing dynamics and to ask if a signature of the original phases remains, even if the transition is lost. The introduction of a spatial bias (a preferred direction) is one factor that is known to affect the scaling functions and the exponents characterizing nonequilibrium transitions. Examples where bias plays a role include models of extremal dynamics [1,2], the simple exclusion process [3], sandpile models [4], directed percolation [5], interface depinning [6], reaction-diffusion systems [7], and random walkers in fractal media [8]. These examples consist of systems in which parameters need to be tuned to reach criticality as well as systems that are self-organized critical. In both cases, bias either changes the universality class characterizing the system [1–7] or causes localization [8] or induces boundary driven transitions [3].

In this paper we study the effect of bias on a recently introduced model of aggregation and fragmentation, which was shown [9,10] to exhibit an unusual nonequilibrium phase transition belonging to a universality class different from models studied earlier [3,11]. We show that bias in this model plays a different role from any of those mentioned above in that even an arbitrarily small bias inhibits the phase transition entirely in one dimension. Remarkably though, a signature of the transition remains and modifies the finite-size behavior of the system, and we characterize the scaling implications of this. In two and higher dimensions, we show that bias has no effect on the phase transition.

We define the model on a d -dimensional hypercubic lattice with periodic boundary conditions. Starting from a ran-

dom distribution of non-negative integer masses, the system evolves in time via the following microscopic processes. In an infinitesimal time dt , (i) with probability pdt (qdt), the mass at each site hops to one of its nearest neighbors with increasing (decreasing) coordinates; (ii) with probability wpd ($wqdt$), unit mass is chipped off from an already existing mass and added to one of its nearest neighbors with increasing (decreasing) coordinates. Following these moves, the masses at each site add up (see Fig. 1). The dynamics conserves the total mass of the system. Hence, the parameters defining the system are the bias $p-q$, the density ρ , and the chipping or fragmentation rate w . The case $p=q$ corresponds to the zero bias case or the symmetric model while the case $p \neq q$ introduces a preferred direction in the motion of the masses and corresponds to the asymmetric model.

The model may be mapped onto generalizations of other

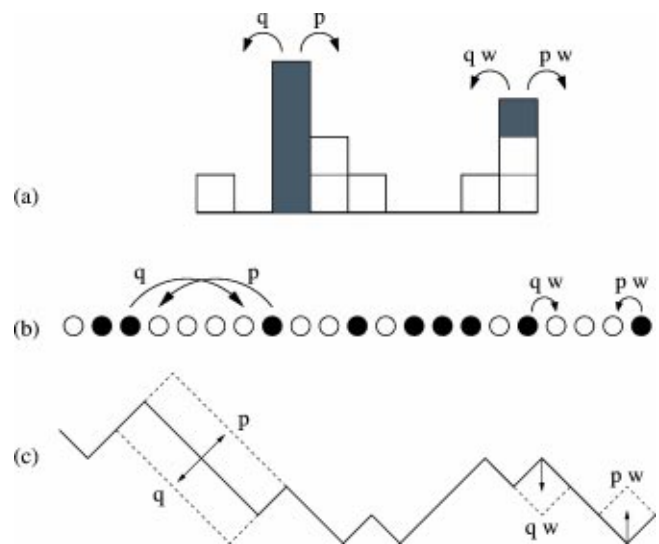


FIG. 1. (a) The model as defined in the text. (b) To obtain the hard core lattice model from the masses, lay down the masses on their sides. (c) The interface is obtained by replacing every shaded circle by a line segment in the $+45^\circ$ direction and every empty circle by a line segment in the -45° direction.

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well studied models of nonequilibrium statistical mechanics. In one dimension the system of masses described above may be equivalently thought of [10] as a collection of particles and holes on a ring, or a one-dimensional interface evolving in time (see Fig. 1). In this language, the $w \rightarrow \infty$ limit exactly corresponds to the well studied symmetric (asymmetric) exclusion process [12] for $p = q$ ($p \neq q$). In the interface language, this corresponds [13] to an interface evolving via the Kardar-Parisi-Zhang or the Edwards-Wilkinson dynamics [14] for the cases $p \neq q$ and $p = q$, respectively. For a finite w , the nearest neighbor particle exchanges of the exclusion process (or corner flips of the interface growth model) are further augmented by long-range moves.

Models similar to the one studied in this paper have also been studied in other contexts. A slightly different off-lattice version of this model was studied within the rate equation approach in the context of polymer chain growth [15]. In this case, the mass clusters were thought to represent polymers. Although the aggregation of polymers and dissociation of single monomers were allowed in this model, it lacked the important process of local diffusion which we include in our study. Various models of coagulation and fragmentation, with coagulation rate proportional to mass, have been studied in the context of gelation [16]. In these systems, due to the enhanced coagulation, a gel (a cluster which contains a finite fraction of the total mass) forms at a finite time. The exponents at the transition point were shown to depend on whether the process of fragmentation was present or absent. Models of fragmentation in which a fraction of existing mass (as opposed to a single particle) may break off have also been studied in [17–19]. In these models, it could be deduced that an infinite aggregate that contains a finite fraction of the total mass never forms. A model with bias very similar in spirit to the model we study here was also studied in the context of traffic flow [20] and the phase transition observed numerically was interpreted as a traffic jam occurring for large densities of cars.

An understanding of the steady state reached by the system may be obtained by considering the limits of only diffusion ($w = 0$) or only chipping ($w = \infty$). Both of these cases are exactly solvable. In the former case, the system maps to the well studied reaction-diffusion system $mA + nA \rightarrow (m+n)A$, and the steady state reached is simply one in which all the particles accumulate on a single site. In the opposite limit ($w \rightarrow \infty$), the model is again exactly solvable and the system reaches a steady state in which the mass is uniformly spread out over the system with the probability that a given site has a mass m being exponentially distributed.

The special case $p = q$, corresponding to zero bias, was studied earlier using a mean field approximation [9], Monte Carlo simulations [10,21], as well as some exact analysis [21]. It was shown that the system undergoes a nonequilibrium phase transition in the ρ - w plane at some critical density $\rho_c(w)$. In particular, $P(m)$, the probability that a randomly chosen site has mass m , was shown to vary for large m as (i) $P(m) \sim \exp(-m/m^*)$ for $\rho < \rho_c$, (ii) $P(m) \sim m^{-\tau}$ at $\rho = \rho_c$ with $\tau = 5/2$, and (iii) $P(m) \sim m^{-\tau} + \text{“infinite aggregate”}$ for $\rho > \rho_c$, where by “infinite aggregate” we mean a single large mass equaling a finite fraction

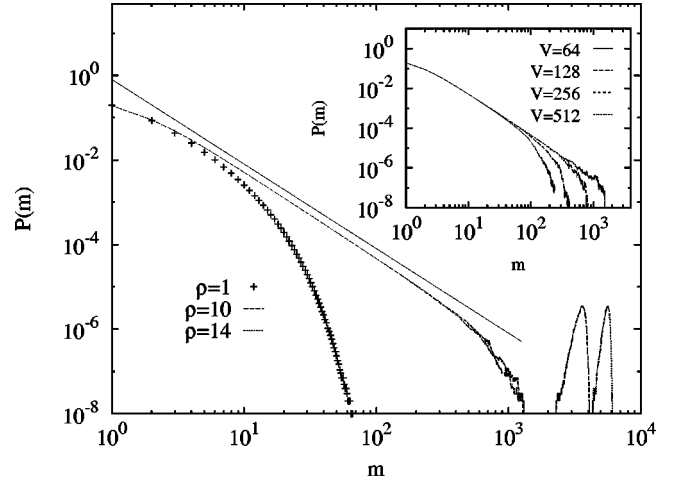


FIG. 2. $P(m)$ is plotted against m for three values of density. The cutoff is seen to depend on ρ for small values of the density, while for larger values it is independent of ρ . The mass in excess of the critical mass forms an aggregate. The solid line has an exponent -2 . The simulation was done for system size $V = 500$ and $w = 3.0$. In the inset, the dependence of the cutoff of the power law on V is shown. The simulations were done for $\rho = 15$.

of the total mass of the system. That is, the mass distribution $P(m)$ changes from an exponential distribution to an algebraic one at ρ_c . For $\rho > \rho_c$, the mass distribution remains the same as at ρ_c , while all the mass in excess of the critical density coagulates together forming an infinite aggregate. The mathematical mechanism giving rise to the infinite aggregate was found to be very similar to that of equilibrium Bose-Einstein condensation in an ideal Bose gas.

Bias is introduced in the motion of the masses on the lattice by letting $p \neq q$. In the limits of only diffusion or only chipping, the steady states reached are the same as those for the $p = q$ case. Hence, one might expect a phase transition in the ρ - w plane as before. Further, the mean field analysis [9,10] does not recognize any difference between the two cases and thus predicts the same behavior as in the zero-bias case. Earlier Monte Carlo simulations of this system in one dimension did indeed seem to suggest the existence of a phase transition similar to the symmetric case, though with $\tau \approx 2.0$ [10]. The results of these simulations are summarized in Fig. 2. For a fixed lattice size and w , when the total mass in the system is increased beyond a certain critical mass, the formation of an aggregate is observed. The power law regime has a lattice size dependent cutoff (see inset of Fig. 2) as in the symmetric case. However, the value of τ (~ 2.0) is a puzzle since a finite system density implies that τ should be strictly greater than 2 (the first moment, which is the density of the system, would diverge if $\tau < 2$).

In this paper, we analyze the model for $p \neq q$ and show that the apparent existence of a phase transition in the Monte Carlo simulations is purely a finite-size effect in one dimension, with the critical density $\rho_c(V)$ for fixed w diverging with system size V as $\ln(V)$. We argue that the exponent τ is exactly 2. In two and higher dimensions, however, a transition from an exponential to an aggregate phase does occur at finite critical density.

The rest of the paper is organized as follows. In Sec. II, we show analytically that an aggregate phase cannot exist in the one dimensional system, thereby showing that there is no phase transition in one dimension. Section III contains numerical evidence for the results of Sec. II, from Monte Carlo simulations of the model. In Sec. IV the exponents describing the probability distribution for mass in one dimension are characterized. Section V contains a numerical study of the two-dimensional problem. Section VI contains a summary and conclusions.

II. ARGUMENTS FOR NO PHASE TRANSITION IN THE PRESENCE OF SPATIAL BIAS IN ONE DIMENSION

In this section, we prove that in the presence of a spatial bias, an aggregate phase cannot be present at any finite density in one dimension. We do so by assuming that an aggregate phase exists, and then showing that this leads to certain contradictions. To proceed, assume that a single infinite aggregate exists in the system and that the rest of the system is at a finite critical density ρ_c (analogous to the symmetric case). Consider now a frame of reference that is attached to this aggregate. Let $\eta_k^\pm(t)$ be the mass transferred in an infinitesimal time dt at time t from a site k lattice sites away from the aggregate to a site $k \pm 1$ sites from the aggregate. Then

$$m_k(t+dt) = m_k(t) + \eta_{k-1}^+ + \eta_{k+1}^- - \eta_k^+ - \eta_k^- + a_k^+ + a_k^-, \quad (1)$$

where $m_k(t)$ is the mass at a site k lattice sites from the aggregate at time t , and a_k is the change in mass due to the diffusion of the aggregate. The time dependence of the variables on the right hand side of Eq. (1) have been suppressed for the sake of clarity. Equation (1) accounts for all the ways in which the mass on a site k sites away from the aggregate can change under the dynamics. These changes are caused either by the mass transfer to and from neighboring sites as exemplified by the η_k^\pm 's or by the motion of the aggregate, which leads to a relabeling of sites, as depicted by the a_k^\pm 's.

From the definition of the model, it is clear that

$$\eta_k^+ = \begin{cases} m_k & \text{with prob } pdt, \\ 1 - \delta_{m_k,0} & \text{with prob } p w dt, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

and similarly for η_k^- , with p replaced by q in Eq. (2). When the aggregate hops, the sites have to be relabeled and this leads to

$$a_k^+ = \begin{cases} m_{k+1} - m_k & \text{with prob } pdt, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

and correspondingly for a_k^- , with $k+1$ replaced by $k-1$ and p by q in Eq. (3). Taking averages on both sides in Eq. (1) and setting the time derivatives to zero in the steady state, we obtain

$$\begin{aligned} \frac{d\rho_k}{dt} = 0 = & w[ps_{k-1} - (p+q)s_k + qs_{k+1}] + (p+q) \\ & \times [\rho_{k-1} - 2\rho_k + \rho_{k+1} + \rho_0(2\delta_{k,0} - \delta_{k,-1} - \delta_{k,1})], \end{aligned} \quad (4)$$

where $s_0 = 1$, and $\rho_k = \langle m_k \rangle$ and $s_k = \langle 1 - \delta_{m_k,0} \rangle$ are the average density and occupation probability of a site k lattice sites away from the aggregate, respectively. A point to note about Eq. (4) is that bias plays a role only in the terms coupling to the chipping rate w . It can be checked that introducing a bias only in the diffusion move and keeping the chipping move symmetric does not change the behavior of the model from the fully symmetric version $p=q$.

We now consider Eq. (4) in the steady state when the time derivative is set to zero. The set of linear equations in Eq. (4) may be solved for a finite or an infinite lattice to obtain the ρ_k 's in terms of the s_k 's. In the former case, a closed form expression for the densities is easily obtained. However, it is more informative to look at the equations for an infinite lattice, where the sites with negative (left of aggregate) and positive (right of aggregate) indices may be treated separately. For this case, we obtain for $n > 0$,

$$\begin{aligned} \rho_n = n \left[\rho_1 + \frac{w}{p+q}(qs_1 - p) + \frac{w(p-q)}{p+q}s_c^r \right] \\ + \frac{w(p-q)}{p+q} \sum_{k=1}^n (s_k - s_c^r) + \frac{wp}{p+q}(1 - s_n), \end{aligned} \quad (5)$$

$$\begin{aligned} \rho_{-n} = n \left[\rho_{-1} + \frac{w}{p+q}(ps_{-1} - q) - \frac{w(p-q)}{p+q}s_c^l \right] \\ + \frac{w(p-q)}{p+q} \sum_{k=-1}^{-n} (s_c^l - s_k) + \frac{wq}{p+q}(1 - s_{-n}), \end{aligned} \quad (6)$$

where $s_c^r = \lim_{n \rightarrow \infty} s_n$ and $s_c^l = \lim_{n \rightarrow \infty} s_{-n}$.

By assumption, we require that ρ_n tends to a finite value as $n \rightarrow \infty$. For this, we require first that the term proportional to n in Eqs. (5) and (6) vanishes and secondly that $s_{\pm n}$ approaches its asymptotic value faster than $1/n$. The first condition expresses ρ_1 and ρ_{-1} in terms of s_1 and s_{-1} as

$$\rho_1 = \frac{w(p - qs_1) - w(p-q)s_c^r}{p+q}, \quad (7)$$

$$\rho_{-1} = \frac{w(q - ps_{-1}) - w(q-p)s_c^l}{p+q}, \quad (8)$$

which when substituted in the $k=0$ equation of Eq. (4) leads to

$$s_c^r = s_c^l \equiv s_c. \quad (9)$$

This is consistent with the fact that far from the aggregate, the occupation probability is the same on either side. The second condition, that $s_{\pm n}$ approaches its asymptotic value

faster than $1/n$, will be useful for determining the exponents and will be studied numerically in Sec. III.

A important point to note from Eq. (5) is that the two cases $p=q$ and $p \neq q$ are quite different. For $p=q$, the cumulative sum over the s_n 's vanishes and ρ_n depends only on the site occupancy of the site n . However, when $p \neq q$, the sum plays a role in determining the value of the site density and it becomes important to understand the behavior of $s_{\pm n}$ as a function of n . We do that in Sec. III.

We now examine the two-point correlations in the presence of the aggregate in order to obtain further relations be-

tween the ρ_k 's and s_k 's. Using these relations we will be able to show a contradiction. Consider the two-point correlations in the aggregate frame of reference. Let

$$C_{r,k} = \langle m_r m_{r+k} \rangle. \quad (10)$$

The equations governing the temporal evolution of the two-point mass-mass correlations may be derived by considering the mass transfer between two neighboring sites [see Eq. (1)]. Multiplying together Eq. (1) for m_r and the corresponding one for m_{r+k} , keeping terms up to order dt , and taking averages, we obtain

$$\begin{aligned} \frac{dC_{r,k}}{dt} = & pC_{r+1,k} + qC_{r+1,k-1} + pC_{r,k-1} + qC_{r,k+1} + pC_{r-1,k+1} + qC_{r-1,k} - 3(p+q)C_{r,k} + w[-pD_{r-1,k+1} + (p+q)D_{r,k} \\ & - qD_{r+1,k-1} - pE_{r,k-1} + (p+q)E_{r,k} - qE_{r,k+1}] - \delta_{k,1}[qC_{r+1,0} + pC_{r,0} + wps_r + wqs_{r+1}], \quad r, k = 1, 2, \dots, \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{dC_{r,0}}{dt} = & (p+q)[C_{r+1,0} - 2C_{r,0} + C_{r-1,0}] + 2pC_{r-1,1} + 2qC_{r,1} - w[2pD_{r-1,1} + 2qE_{r,1}] + w[(p+q)s_r + ps_{r-1} + qs_{r+1}], \\ & r = 1, 2, \dots, \end{aligned} \quad (12)$$

where

$$D_{r,k} = \langle \delta_{m_r,0} m_{r+k} \rangle, \quad k = 1, 2, \dots, \quad (13)$$

$$E_{r,k} = \langle m_r \delta_{m_{r+k},0} \rangle, \quad k = 1, 2, \dots \quad (14)$$

Also, for Eqs. (11) and (12) to be valid when $r=1$, we need to set $C_{0,k} = D_{0,k} = 0$. In the steady state, the time derivatives can be set to zero. Summing Eq. (11) over all $r, k = 1, 2, \dots$, and subtracting from this $1/2$ times the sum over $r=1, 2, \dots$ in Eq. (12), we obtain

$$(p+q)C_{1,0} = wp - 2(p+q) \sum_{k=1}^{\infty} C_{1,k} + 2wq \sum_{k=1}^{\infty} D_{1,k} + wqs_1. \quad (15)$$

The left hand side of Eq. (15) is a finite number while the right hand side has two infinite sums. These two sums can add up to give a finite value only if asymptotically the terms in the two summations are equal. Using the fact that the two-point correlations decouple when the separation between the two points become large, we obtain

$$(p+q)\rho_1 = wq(1-s_1). \quad (16)$$

Equations (16) and (7) have to be simultaneously satisfied. This is possible only if

$$w(p-q)(1-s_c) = 0. \quad (17)$$

The fragmenting rate w being equal to zero is the trivial limit in which the entire mass in the system coagulates together to form an infinite aggregate. When there is a bias $p \neq q$, then for finite w the only way Eq. (17) can be satisfied is if s_c

$= 1$; i.e., the occupation probability far away from the aggregate is 1. This can occur only if $\rho_c = \infty$, which contradicts our initial assumption that ρ_c is finite. Another way of seeing a contradiction is to consider Eq. (5) when $n \rightarrow \infty$. Setting $s_c = 1$, we obtain that the densities far away from the aggregate become negative for $p > q$. Thus, for any finite w and ρ we always have a contradiction and hence our initial assumption of an aggregate existing at finite density is proved wrong. This proves that for any finite ρ and w , an aggregate phase does not exist in one dimension in the thermodynamic limit. For the symmetric problem ($p=q$), there is no contradiction between Eqs. (16) and (17), since Eq. (17) is automatically satisfied.

III. NUMERICAL CHECKS IN ONE DIMENSION

The results of the previous section thus show that an infinite aggregate cannot exist in an infinite system when there is a nonzero bias. However, numerical simulations of finite-size systems do point to the existence of an aggregate phase (see Fig. 2). Nevertheless, there is no contradiction with the results of Sec. II, provided the critical density $\rho_c(V)$ diverges with the system size V . In this section, we study numerically the system size dependence of $\rho_c(V)$ in one dimension.

In earlier studies [10], the system size dependence of $\rho_c(V)$ was not investigated because there was no systematic way of making an accurate numerical measurement of the critical density. To measure $\rho_c(V)$, we adopted the following procedure. For a fixed lattice size, we start the system with a density much higher than that required to form an aggregate. The system is then allowed to reach the steady state and the biggest cluster is identified as the aggregate. We then measure the density in the rest of the system (excluding the ag-

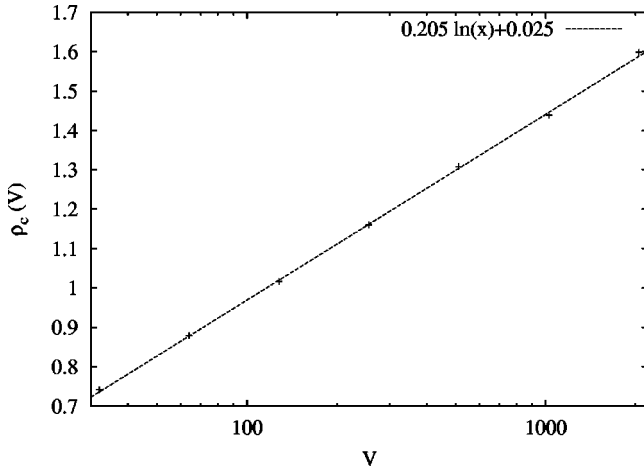


FIG. 3. The critical density $\rho_c(V)$ diverges with the system size V as $\ln(V)$ in one dimension. The simulations were done for the fully asymmetric model $p=1$ and $q=0$.

gregate) and use the fact that the state of the rest of the system resembles that at criticality. In Fig. 3, the system size dependence of $\rho_c(V)$ is shown on a semilogarithmic scale. From the numerical evidence, we conclude that $\rho_c(V) \sim \ln(V)$.

As a further check, we study the occupation probability numerically. From Eq. (5), the dependence of ρ_n on $\Sigma(s_{\pm k} - s_c)$, taken together with the fact that $\rho_c(V) \sim \ln(V)$, implies that

$$|s_{\pm k} - s_c| \sim \frac{a_{\pm}}{|k|^x}, \quad k \gg 1, \quad (18)$$

with $x=1$, which in turn implies that $s_c(V)$ converges to its asymptotic value as $1/V$. Both of these requirements are consistent with numerical simulations (see Fig. 4). The simulations were done for the fully asymmetric model $p=1$ and $q=0$.

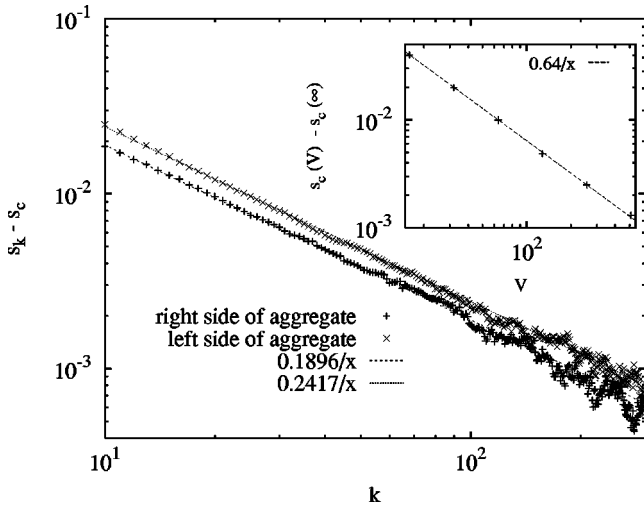


FIG. 4. s_k converges to s_c (≈ 0.2775 in this case) as $1/k$. The simulations were done for lattice size 4000 and $w=1.0$. The inset shows the finite-size correction to s_c .

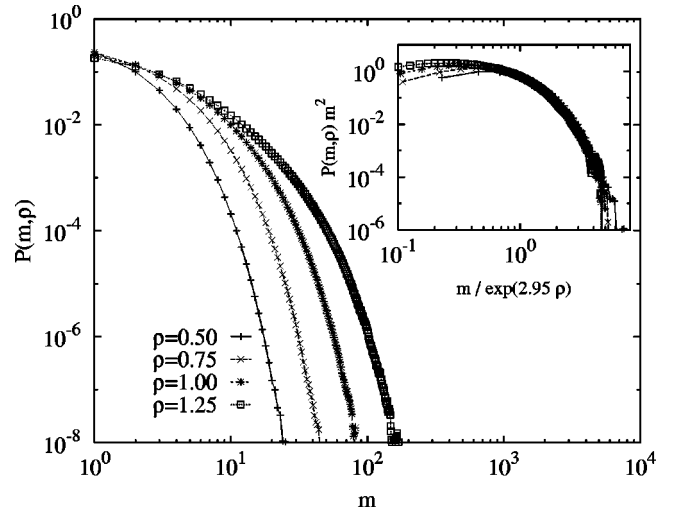


FIG. 5. The probability distribution for mass for different values of ρ . The inset shows the scaling plots when the distributions are scaled as in Eq. (20).

IV. PROBABILITY DISTRIBUTION

From the analytical and numerical evidence of Secs. II and III it is clear that the system is sensitive to the manner in which the limits $M \rightarrow \infty$ and $V \rightarrow \infty$ are taken, where M is the total mass in the system. When M is increased beyond M_c ($= \rho_c V$) keeping V fixed, an infinite aggregate does form in the system. In this regime, in analogy with the symmetric problem [21], we can then write a scaling form for the probability distribution for $M > M_c$ as

$$P(m, V) = \frac{1}{m^\tau} f\left(\frac{m}{V^\phi}\right) + \frac{1}{V} \delta(m - (M - M_c)). \quad (19)$$

Since the mean mass in the power law part of the distribution scales as $\ln(V)$ (see Fig. 3), we immediately derive $\tau=2$. In addition, from the consideration that there is only one aggregate, the two exponents τ and ϕ are known to obey the scaling relation $\phi(\tau-1)=1$ [21]. This implies that $\phi=1$. The fact that the cutoff of the power law distribution scales as V , and not as a smaller power of V as in the symmetric case [21], is consistent with the fact that the transition does not exist for large V .

What is the behavior of the system when the order of limits is reversed and $M, V \rightarrow \infty$ keeping $\rho = M/V$ fixed? In this case, we make the reasonable ansatz

$$\lim_{V \rightarrow \infty} P(m, \rho) = \frac{1}{m^\tau} g\left(\frac{m}{m^*}\right), \quad (20)$$

with the same exponent $\tau=2$. The requirement that $\langle m \rangle = \rho$ taken together with Eq. (20) implies that the cutoff $m^* \sim e^{\alpha\rho}$. Scaling plots of the probability distribution for various values of ρ scaled as in Eq. (20) are shown in Fig. 5.

From simulations, it is seen that the function $g(x) \sim \text{const}$ when $x \rightarrow 0$ (see inset of Fig. 5). This taken together with the exponential divergence of the mass cutoff implies that, for infinite ρ , $P(m)$ is a pure power law. This is similar

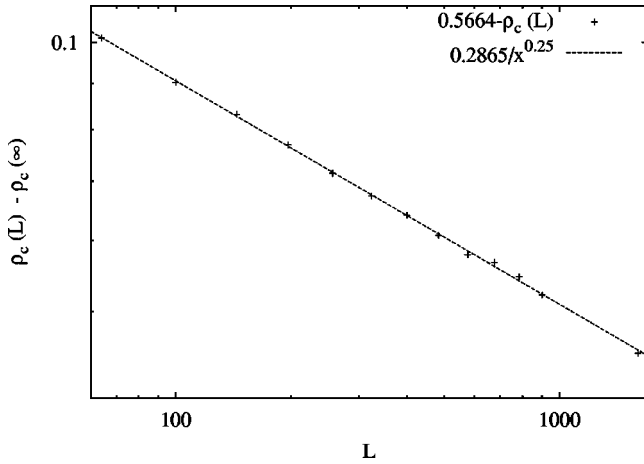


FIG. 6. The convergence of the critical density to a finite value with increasing system size in two dimensions is shown. The simulations were done for the fully asymmetric model $p=1$, $q=0$ and the fragmenting rate $w=1.0$.

to the steady state of the Takayasu model [22] for river networks where mass aggregates in the presence of a constant influx of particles, and the steady state has a nontrivial power law distribution.

V. NUMERICAL SIMULATIONS IN TWO DIMENSIONS

The arguments used in Sec. II to prove that there is no phase transition in one dimension are very specific to one dimension and cannot be extended to two and higher dimensions. Instead, in this section, we numerically study the model in two dimensions. First, we measure the critical density $\rho_c(V)$ for lattice sizes varying from 8×8 to 32×32 using the same method as that used for the one-dimensional simulations. In this case it is seen that $\rho_c(V)$ does converge to a finite value when the system size is extrapolated to infinity (see Fig. 6). Hence, a phase transition does exist in the infinite-system limit.

We now show that for the system with spatial bias the value of the power law exponent τ in two dimensions is close to that in the zero-bias case. It is difficult to make a direct measurement of τ because the cutoff to the power law grows slowly with the system size and is not large enough (cutoff ~ 200 when the system size ~ 2000) for an accurate measurement of the slope. For instance, in the zero-bias case direct measurements of τ [9,10] gave a value close to 2.3 while indirect numerical methods [21] showed that τ is close to $5/2$ in all dimensions. Therefore, rather than measure τ directly for the asymmetric model, we compare the simulation results (see Fig. 7) for the fully asymmetric problem with those for the symmetric problem with the same parameters. The slopes of the two curves are comparable. Hence, we conclude that the exponent τ for the asymmetric model in two dimensions is very close to $5/2$, as in the symmetric model.

VI. SUMMARY AND CONCLUSIONS

In summary, we have investigated in detail the effect of introducing a spatial bias on the nonequilibrium phase tran-

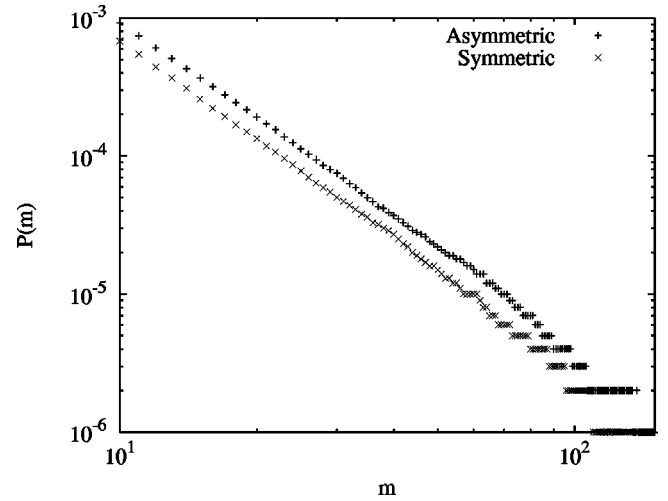


FIG. 7. The value of the power law exponent τ for the fully asymmetric model in two dimensions is very close to that of the symmetric model in two dimensions. The simulations were done for a 30×30 lattice with $w=1.0$ and $\rho=10.0$.

sition in a model of coagulation and fragmentation. We show analytically that the phase transition is inhibited in one dimension. However, a signature of the two original phases remains and the scaling implications of this are characterized. We have also resolved the puzzle of the exponent τ being very close to 2. In two dimensions, the phase transition is shown numerically to exist.

We now give a more intuitive explanation of why the phase transition is curbed in one dimension but not in two dimensions. In this model, there are two competing processes. While the diffusion move creates larger and larger masses by coagulation, the fragmentation move tends to create smaller masses as well as inhibit the formation of large masses. If the diffusion move were to be considered by itself, then a cluster of size l would be created in time of order l^2 . In one dimension, if the fragmentation move were to be considered by itself, then a fluctuation of order l would be dissipated in time of order $l^{3/2}$ for the asymmetric model and of order l^2 for the symmetric model. These exponents are known exactly because of the existing exact analogy in one dimension between only fragmentation with (without) bias and the asymmetric (symmetric) exclusion process. Thus, for the asymmetric problem, fragmentation always wins out over diffusion and we only have an exponential phase. However, in two dimensions, bias is irrelevant for the fragmentation move and hence a fluctuation of order l gets dissipated in time of order l^2 , which is of the same order as the time required to create a cluster of size l by diffusion.

To carry this argument further, we can study the symmetric problem by slowing down the diffusion rate. This can be done by introducing a mass dependent diffusion rate $\sim m^{-\alpha}$ with $\alpha > 0$. The above arguments would then imply that this dynamics ought not to have a phase transition for any $\alpha > 0$. This is indeed the case, and it can be shown that the phase transition does become curbed in all dimensions [23], as predicted.

There remain several interesting questions to investigate

further. While we have numerically shown that $\tau=2.0$, it would be interesting if it could be derived from first principles by solving the model. Further, for an infinite system, the probability distribution of the masses has the form [Eq. (20)]

$$P(m) \sim \frac{1}{m^2} e^{-\beta m e^{-\alpha \rho}}, \quad m \gg 1,$$

where β could depend on ρ and w while α depends only on w . The origin of the length scale $e^{\alpha \rho}$ under this dynamics is an interesting point that remains to be understood. Also,

since the phase transition in one dimension is a finite-size effect, the implications of the traffic jam that was seen in [20] need to be reexamined.

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